Revision work done CHECKLIST:

**Introduction**:

* Remove paragraph on temperature performance. <- not referenced and mentioned any further in the text.
* Substitution of the word “chaotic” with “non-deterministic” throughout the text in order to avoid confusion and chaos theory (as recommended by Reviewer 2).
* Added a sentence relating our results to the publication [T. S. Mansuripur et. al., “*Singlemode instability in standing-wave lasers: The quantum cascade laser as a self-pumped parametric oscillator*,” Phys. Rev. A, vol. 94, p. 063807, 2016*],* which is highly relevant to our work but was only discovered by the authors AFTER the submission was made. Consecutively we also added a new citation to this publication.
  + Before:
    - We further show that upon suppression of SHB, one can recover the comb character of the laser, albeit over a reduced spectral bandwidth.
  + After:
    - We further show that upon suppression of SHB, one can recover the comb character of the laser, albeit over a reduced spectral bandwidth. This result also corresponds to recent experimental findings affirming that single-mode instabilities, introduced by spatial hole burning, yield an incoherent "dense" emission spectrum, when the QCL is driven a little above threshold [12].

**Model**:

* Defined, and *q*0 as suggested by Reviewer 2. Paragraph 5 of this section was modified accordingly:
  + Before:
    - Further, we have denoted with *n* the background refractive index, with *c* the velocity of light in vacuum and with …
  + After:
    - Further, denotes the 1' 3 detuning, *n* the background refractive index, *c* the velocity of light in vacuum, *q*0 the elementary charge and…”

**Mode proliferation mechanisms**:

* Changed the colours in Fig. 1 to be more suitable for grayscale printing.
* Modified paragraph 4 from this section to clarify the choice of bias (11 kV/cm) in our simulations. This modification was necessitated by several comments from Reviewer 1, who inquired about the dependence of the power distribution of the lasing modes on the applied bias. The corresponding sentence was modified as follows:
  + Before:
    - In both cases, the simulations were based on the THz QCL from [4], biased at 11 kV/cm.
  + After:
    - In both cases, the simulations were based on the THz QCL from [4], biased at 11 kV/cm, which is approximately the bias aligning the injector and the upper laser levels, i.e. [11].

**Comb degradation mechanisms**:

* Modified a sentence in paragraph 6 of the text due to a semantic error.
  + Before:
    - In free-running QCLs, due to their broadband nature and ultrafast carrier dynamics [12], there will be a strong competition between FWM and dispersion, the outcome of which will determine the free spectral range
  + After:
    - In free-running QCLs, due to their broadband nature and ultrafast carrier dynamics [9], there will be a strong competition between FWM and dispersion, the outcome of which will determine the emission spectrum of the device.
* Changed the colours in Fig. 2 to be more suitable for grayscale printing.

**Time evolution of the multimode spectrum:**

* Replot Fig. 3,4 and 5 with the optical power plots in log-scale, as recommended by Reviewer 1.
* Add corresponding beatnotes to figure 3 for ease of comparison with figures 4 and 5 and modified the caption accordingly.
* Modified paragraph 3 of this section in order to resolve the confusions implied by Reviewer 1 regarding why the lasing modes in the high frequency part of the spectrum are stronger than those in the lower part of Fig. 3. Detailed theoretical explanation is included into the appendix.
  + Before:
    - The results are summarized in Fig. 3. A quick comparison …
  + After:
    - The results are summarized in Fig. 3. Note that for these simulations the laser was biased at 10.8 kV/cm, which is when the injector level is energetically below the upper laser level (i.e. < 0). In Appendix A we show how this leads to lasing predominantly of the high frequency part of the spectrum, in comparison to the case when the laser is biased at 1’3 resonance, when both spectral lobes are approximately equally strong. A quick comparison …
* Remove the last paragraph of this section due to the fact that the appearance of a frequency-doubled beatnote in Fig. 5, is not relevant to the comparisons made to the rest of the simulation!
  + Before:
    - Secondly, in Fig. 5(c) we can notice a very strong and narrow beatnote at the second harmonic of the linear cavity’s round trip frequency frt at approximately 15.8 GHz. This is also not so surprising, since due to the unidirectionality of the field propagation, only contributions from the left-to-right propagating signal are sufficiently strong, and thus the effective round trip time is halved.
  + After:
    - XXX

**Appendix A:**

* Added a dedicated appendix onto the bias dependence of the dominant lobe in the emission spectra. The addition of this section was necessitated by several comments made by Reviewer 1.